GUIDELINES IN TEACHING, LEARNING AND ASSESSMENT FOR MATHEMATICS

(As outlined in the EDUCATORS' GUIDE FOR PEDAGOGY AND ASSESSMENT for Mathematics)

Teaching and Learning

The LAP (Learning and Assessment Programme), is intended to create an atmosphere where learners develop their own problem-solving skills and their ability to think and reason logically; reflect on outcomes and consequences and explore possible alternatives and apply interesting and realistic contexts that are personally meaningful to them. With the use of LAPs, teachers will be encouraged to create situations and resources which are intrinsically interesting, culturally embedded and cognitively engaging and enable learners to connect the various types of information that they have acquired.[1]

The dual nature of mathematics – mathematics as a body of knowledge (a product) and mathematics as a mode of enquiry (a process) – makes it imperative that educators use a variety of teaching approaches to cater for both these aspects. Effective teachers of Mathematics recognise that there is no single best way to teach Mathematics and adopt those teaching approaches which will help their learners to learn the subject and to appreciate its different facets. Irrespective of the approach used, the learner should remain at the centre of the learning experience. Ultimately, it is what learners learn that is important.[2]

Learners will be best served if during the scholastic year educators make use of the following three teaching approaches:

i) Teaching through exposition

This approach normally involves the teacher in stating the objectives of the lesson. This is then followed by an explanation during which the teacher introduces the new knowledge. This new learning is consolidated by setting learners a number of tasks aimed at helping learners consolidating and practising their newly learnt knowledge. The underlying psychology of learning is a behaviourist one. This approach helps learners learn new facts (e.g. terms, notation and conventions) and skills (e.g. practical measurement skills, calculator skills, computer skills and communication skills). It is also helpful in introducing new concepts and in linking these concepts within conceptual structures and in presenting strategies (e.g. problem-solving, investigating, estimating, approximating, reasoning, and proving and disproving).

ii) Teaching through discovery

This approach requires the teacher to set learners tasks in which they discover some new mathematical knowledge. In these tasks learners collect, process and analyse data to obtain information, or examine a few particular examples in order to identify common properties. In these tasks learners are also encouraged to make and test relevant hypotheses and/or make generalisations. The underlying psychology of learning is a constructivist one. This approach is suitable for facilitating learners' further development of concepts and conceptual structures; facts

[1] Educators' Guide for Pedagogy and Assessment (2015), page 6.
[2] Educators' Guide for Pedagogy and Assessment (2015), page 40.

(e.g. terms, notation and conventions), skills (e.g. practical measurement skills, calculator skills, computer skills and communication skills) and strategies (e.g. problem solving, investigating, estimating, approximating, reasoning, and proving and disproving) as well as in improving learners' personal qualities (e.g. being imaginative, creative, flexible, persistent, systematic, independent; cooperative) and their attitudes towards mathematics (e.g. interest, motivation, pleasure and enjoyment).

iii) Teaching through exploration

This approach usually consists of the teacher setting learners tasks in which learners test a hypothesis based on their understanding of a concept. These tasks might be open or closed problems or might include investigations. In these tasks learners must collect further data and process it to obtain further information required to complete their tasks. The underlying psychology of learning is a constructivist one. This approach is appropriate for facilitating learners' use of facts (e.g. terms, notation and conventions), skills (e.g. practical measurement skills, calculator skills, computer skills and communication skills), concepts and strategies (e.g. problem-solving, investigating, estimating, approximating, reasoning, and proving and disproving). It can also have a beneficial effect on learners' personal qualities and their attitudes towards mathematics (e.g. interest, motivation, pleasure and enjoyment).[3]

Although not every learner will learn mathematics with the same ease and at the same pace, educators can greatly help their learners' efforts to learn mathematics if they structure their teaching to focus on three key elements[4]:

i. automatic recall of facts;
ii. procedural fluency;
iii. conceptual understanding.

All three of these elements are needed for the successful learning of mathematics.

Factual knowledge of basic mathematical facts (e.g. multiplication facts) is needed because rapid and relatively attention free recall of these facts is crucial when solving complex mathematical problems. The ability to perform a small set of algorithms is also critical in problem solving because complex mathematical problems can often be broken down into simpler mathematical problems that require the correct performance of some frequently encountered routine mathematical procedure (e.g. algorithm to multiply two 2-digit numbers; algorithm to multiply two trinomials; algorithm to solve a quadratic equation).

Conceptual knowledge is also required when solving complex mathematical problems as understanding is at the very core of problem solving. Unless learners understand what they are doing, it is highly unlikely that they will be able to make sense of the problem and what is required to solve it. Moreover, the correct interpretation of a solution requires understanding of meaning: knowing why and not simply knowing that or knowing how (e.g. why a negative answer might be meaningless as a solution to a given problem).

Whatever teaching approach is adopted at any point in time it should aim to help learners develop:

[3] Educators' Guide for Pedagogy and Assessment (2015), page 41.
[4] National Research Council, 2001; National Mathematics Advisory Panel (2008), page 11

- A deep understanding of mathematics that is based on knowledge of facts, procedures and meaning.
- The ability to use and apply their mathematical knowledge and understanding to solve a wide range of standard and non-standard problems ideally related to real life situations.
- An understanding and appreciation of the role and purpose of mathematics in our culture and society through appropriate references to the history of the subject.
- The ability to think and communicate mathematically − precisely, logically, creatively and effectively.
- A positive attitude towards mathematics that fosters creativity, confidence, perseverance and enjoyment of the subject.
- The ability to work both independently, collaboratively and cooperatively when doing Mathematics.
- A secure foundation for the further study of mathematics.
- An appreciation of the interdependence of the different strands and topics of mathematics.
- An appreciation of the interdisciplinary nature of mathematics and its use in other areas of knowledge.
- The ability to make efficient, creative and effective use of appropriate technology in mathematics.

(Adapted from Educators' Guide for Pedagogy and Assessment, 2015, page 58)

To support curriculum planning and to ensure that all learners have access to an active, enterprising learning environment, a coherent approach to planning learning, teaching and assessment and to sharing information about progress and achievements is needed. In undertaking this type of curriculum planning, it is important not to see the Subject Learning Outcomes (SLOs) as limiting factors containing the learning potential of learners and preventing any deviation of learning beyond that contained within the SLOs.

How, where and when the outcomes are taught and learned is at the discretion of the educator. The SLOs are there to demystify the assessment process by setting out straightforward learning expectations. In doing so, assessment is bound to evidencing the meeting of these same expectations. Once the learning expectations are set educators can begin to introduce the flexibility in curriculum design and delivery that has been difficult to do up to this point.

The learning outcomes approach allows educators to lean towards learner-centric teaching and learning strategies. This will mean knowing the many ways in which learners are different from one another, which of the many ways of learning are significant to the learning at hand and how to deal with this variance in ways that are supportive of the individual learners and allow them to progress. [5]

Assessment

The summative assessment of a learner's performance should be based on formal one-off as well as ongoing assessment methods.

Examples of the latter include:

- Schoolwork/homework
- Tasks including Investigations (e.g. Investigate the effect of the gradient on a linear graph)

[5] Educators' Guide for Pedagogy and Assessment (2015), page 42.

- Class presentations
- Reports of class experiments (e.g. dice experiments in probability)
- Take home projects (e.g. producing a chart with tessellating shapes)
- Surveys (e.g. colour of cars that pass in front of the learner's home)
- Outdoor field activity write-ups (e.g. mathematics trails)
- Cross-curricular activity reports (e.g. reporting on the mathematics involved in preparing a healthy meal activity within Home Economics reporting on the mathematics involved in a trekking activity using a compass within Physical Education)
- Portfolios
- Journals [6]

Formative assessment

It is important that teachers do not content themselves with simply obtaining data on what their learners know and can do at the end of a learning stage but that they use this data to identify the learning difficulties of their learners and to adjust the teaching and learning experiences in their classrooms accordingly. As such, data collected for summative purposes should also be used for formative purposes.

Other useful information on the effectiveness of teaching can be gathered daily by means of the following informal assessment methods:

• Observations of learners' working – individually, cooperatively and collaboratively

• Questions – open and closed teacher-learner, learner-learner and learner-teacher

• Discussions – online and face to face teacher-learner and learner-learner

• Classroom tasks – open and closed routine and non-routine practical and non-practical

• Quizzes

• Games

Importance of Peer and Self-Assessment

In an age where lifelong learning is becoming the norm, learning to assess one's own performance in an effective and honest manner is a vital skill. It helps the learners of today become effective and successful learners of tomorrow. For this reason, it is important that learners take responsibility for their own learning and contribute to the learning of their colleagues by using success criteria to assess their own learning and their peers'.

Importance of Problem-solving and Investigating

Given that problem-solving and investigating are at the heart of mathematics it is expected that the process of assessment will produce appropriate feedback on the teaching and learning process if it focuses on the learners' abilities to understand a problem and/or investigation; to select an appropriate strategy; to apply suitable conceptual, procedural and factual knowledge to solve it;

[6] Educators' Guide for Pedagogy and Assessment (2015), pages 56-57.

and to verify and interpret the resulting answer. Thus, it is recommended that the majority of assessment tasks should focus on problem-solving and investigating.

Importance of Communicating and Reasoning

The ability to communicate mathematics through oral, written and visual means is of paramount importance, as is the ability to conjecture, think and prove mathematically and so the ability to communicate and the ability to reason are critical components of any assessment process in the mathematics classroom.[7]

Reporting

Evidence informing reporting should be drawn from a range of sources, including formal and informal assessment interventions, and educators should apply their professional judgement to a sufficient and robust body of evidence that allows them to report with confidence about progress made against a significant body of learning. Formal summative assessment interventions need to be subject to collaborative design and development and feature a measure of quality assurance and moderation to ensure what is reported is benchmarked against a wider understanding of the national standard.[8]

References

Ministry for Education and Employment, 2015. *Educators' Guide for Pedagogy and Assessment for Mathematics.* Available at: https://www.schoolslearningoutcomes.edu.mt/files/documents/01_Mathematics.144501456314.pdf [Accessed on 10 August 2021]

US Department of Education, 2008. *National Mathematics Advisory Panel, Final Report*. Available at: https://files.eric.ed.gov/fulltext/ED500486.pdf [Accessed on 10 August 2021]

[7] Educators' Guide for Pedagogy and Assessment (2015), page 57.
[8] Educators' Guide for Pedagogy and Assessment (2015), page 69.